**Exercise Set 4.1**

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a. Yes: −17 is an odd integer.  
b. Yes: 0 is an even integer.  
c. Yes: 2k−1 is an odd integer.

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a) 6m + 8n is even? TRUE  
b) 10mn + 7 is odd? TRUE  
c) If m > n > 0, is - composite? FALSE

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Description automatically generated

a. Yes: 4rs is even.  
b. Yes: 6r+4s2+3 is odd.  
c. Yes: r2+2rs+s2 is composite.



Let a = −3 and b = 0. Then a= 0.

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2 = +   
4 =   
6 = + +   
8 = +   
10 = +   
12 = + +   
14= + +   
16 =   
18 = + +  
20 = +   
22 = + +  
24 = + +

**Exercise Set 4.2**

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A number is rational if it is a ratio of two integers, of which the second (the denominator) is not zero. By the closure property of the integers under subtraction, a−b must be an integer, since a and b are both integers. By the zero product property,  = b × b ≠ 0, because b ≠ 0. Applying the zero product property again, a≠ 0, because a ≠ 0 and

≠ 0. Finally, by the closure property of the integers under multiplication, a is an integer since a and b are both integers, so  is a ratio of two integers, the second of which is not zero. Therefore,  is a rational number by definition.

**Exercise Set 4.3**

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4.

Yes, 3 divides (3k + 1)(3k + 2)(3k + 3). To see this, note that 3k + 3 = 3(k + 1) and that 3|3(k + 1), since 3(k + 1) = 3 × (k + 1), where k + 1 is an integer because k and 1 are both integers. By Theorem 4.3.3 (transitivity of divisibility), since 3 | ( 3k + 3) and (3k + 3) | (3k + 1)(3k + 2)(3k + 3),it must be that 3|(3k + 1)(3k + 2)(3k + 3).

5.

Yes, 6m(2m + 10) is divisible by 4. To see this, note that 6m(2m + 10) = 12m (m + 5)

= 4(3m) (m + 5). Clearly, 4(3m)(m + 5) is divisible by 4, so we get that 4 × (3m)(m + 5) = 6m(2m + 10), so 4 divides 6m(2m + 10) by definition.

15.

Let a, b, and c be integers such that a | b and a | c. Then by the definition of divisibility, b=ma and c = na for some integers m and n. But this means that b+c=ma + na = (m + n)a, where m + n is an integer because m and n are integers. Therefore, by the definition of divisibility, a | (b + c).

**Exercise Set 4.4**

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1. q = 7, r = 7

n = dq + r

70 = 9(7) + 7

2. q = 8, r = 6

n = dq + r

62 = 7(8) + 6

3. q = 0, r = 36

n = dq+r

36 = 40(0) + 36

4. q = 0, r = 3

n = dq + r

3 = 11(0) + 3

5. q = -5, r = 10

n = dq+r

−45 = 11(−5) + 10

6. q = -4, r = 5

n = dq + r

−27 = 8(−4) + 5

**Exercise Set 4.5**



This is true. Let x be any real number. Then ⌊x − 1⌋ is the unique integer n such that

n ≤ x – 1.

**Exercise Set 4.6**

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3. We prove by contradiction. Suppose that 3 | (3n + 2) for some integer n. Then by the definition of "divides," 3n + 2=3k for some integer k. Subtracting 3k and 2 from both sides, we get 3n − 3k = −2. Dividing both sides by 3, we get n – k = −23. But this is a contradiction, since the integers are closed under substraction, but −23 is not an integer. Hence, our assumption must be false, and we conclude that, for all integers n, 3n + 2 is not divisible by 3.

10. Suppose there exists some irrational number such that its square root is rational. Then, the square root can be expressed as ab for positive integers a and b. Then, the number is a2b2. However, both the numerator and denominator of this number are integers, as they are each the square of an integer, so the number must be rational, but this is a contradiction, so the statement is true.